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ON THE DYNAMICAL SYMMETRY BREAKING OF THE ELECTROWEAK INTERACTIONS BY THE TOP QUARK

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Abstract: We discuss the electroweak gauge symmetry breaking triggered by a new strong attractive interaction to condensate fermion-antifermion, and topcolor is a prototype. To deal with the fermion pairing, a general method based on the Hubbard-Stratonovich transformation in the functional integral approach is used.

We derive a formula which relates the $W^{\pm}$, Z^0 weak boson masses to that of the condensated fermion, thus generalizing the Pagels-Stokar formula obtained in QCD. The custodial $SU(2)$ electroweak symmetry turns out to be systematically violated, the deviation of $\rho \equiv M_W^2/(M_Z^2 \cos^2\theta_W)$ from unity is related to the new physics scale Λ. Some phenomenological consequences of the top-pair condensation models are discussed. Distinctive signatures of the $\bar{t}t$ scalar bound state, a Higgs boson like denoted by H_t, are the dominant decay modes $H_t \to \Upsilon + \gamma$, $H_t \to \Upsilon + Z^0$, and $H_t \to B^* + \overline{B}^*$.

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The well-known Higgs mechanism[1, 2, 3] that involves an elementary scalar field may not be the unique scenario to spontaneously break the gauge symmetry of the standard electroweak theory[4]. A dynamical symmetry breaking (DSB) due to the condensation of some fermion-antifermion pair may also generate masses to the gauge bosons, a typical example borrowed from superconductivity is the Cooper electron pair. Another example is the nonzero vacuum expectation value of massless quark-antiquark pair, its condensate breaks the QCD chiral symmetry. In all cases, nonzero numbers must be assumed to break the symmetries, i.e. $\langle 0|\ \Phi\ |0\rangle \neq 0$, $\langle 0|\ \mathrm{e}^-\ \mathrm{e}^-\ |0\rangle \neq 0$, $\langle 0|\ \overline{q}q\ |0\rangle \neq 0$ respectively in the standard Higgs mechanism, the Cooper electron pair in superconductivity, and quark-antiquark pair in QCD.

With DSB, in order to have large values for the $\mathrm{W}^{\pm}$ and Z^0 weak boson masses, there must exist beyond the standard model some new attractive interaction with sufficiently massive fermions involved which replaces the Higgs potential $\lambda\Phi^4 + \mu^2\Phi^2$ with the wrong sign $\mu^2 < 0$. This idea has motivated the topcolor interaction[5, 6, 7, 8] and its extension[9, 10, 11] as the dynamical breaking of the electroweak symmetry due to the condensation of the top-antitop pair since the top quark is the most massive elementary particles. A review of the top-condensation models with extensive references is recently available[12].

An attempt is made in this note to establish a relation between the masses of the condensated fermion and the weak vector bosons $W^{\pm}$, Z^0. The Hubbard-Stratonovich transformation[13] applied to the functional integral method, previously developped by one of us[14] for condensed matter physics, turns out to be particularly powerful for treating the problem of fermion pairing considered here.

We start by introduce a system of left-handed top and bottom quarks put into an SU(2) doublet:

$$\psi_{\mathrm{a}},\ \mathrm{a} = 1, 2\ \ \text{with}\ \ \psi_1(\mathrm{x}) = 1 - \gamma_5 2\psi_{\mathrm{b}}(\mathrm{x})\,, \quad \psi_2(\mathrm{x}) = 1 - \gamma_5 2\psi_{\mathrm{t}}(\mathrm{x})\ , \tag{1}$$

and a singlet fermion χ which is the right-handed top quark,

$$\chi(x) = 1 + \gamma_5 2\psi_t(x)\ . \tag{2}$$

We further postulate that the effective four-fermion topcolor interaction – mediated by topgluons $\mathcal{G}_t$ – may be written in the form

$$\mathcal{L}_{int} = \chi_\delta(x)\overline{\psi}^{\mathrm{a},\alpha}(x)V^{\delta\beta}_{\alpha\gamma}\psi_{\mathrm{a},\beta}(x)\overline{\chi}^{\gamma}(x)\ , \tag{3}$$

$$V^{\delta\beta}_{\alpha\gamma} = G(\gamma_\mu)^{\beta}_{\alpha}(\gamma^{\mu})^{\delta}_{\gamma} \tag{4}$$

with some strong coupling constant G having the (mass)$^{-2}$ dimension. The quark color index is implicitly understood, however it is convenient to explicit the spinor indices $\alpha, \cdots, \delta = 1\cdots 4$. Starting from massless fields, the role of the topcolor interaction (4) is to dynamically generate masses to both the top quark as well as to the gauge bosons. On general ground, one would expect that these masses are functions of the coupling constant G and of the *new physics* energy scale Λ, as we will see later. We consider now the functional integral of the system

$$Z = \int [\mathrm{D}\psi]\ [\mathrm{D}\overline{\psi}]\ [\mathrm{D}\chi]\ [\mathrm{D}\overline{\chi}]\ \exp\left\{\mathrm{i}\int \mathrm{d}^4\mathrm{x}\left[\overline{\psi}^{\mathrm{a},\alpha}(\mathrm{x})(\partial\!\!\!/)^{\beta}_{\alpha}\psi_{\mathrm{a},\beta}(\mathrm{x}) + \overline{\chi}^{\alpha}(\mathrm{x})(\partial\!\!\!/)^{\beta}_{\alpha}\chi_{\beta}(\mathrm{x})\right]\right\}$$

$$\times \exp\left\{\mathrm{i}\int \mathrm{d}^4\mathrm{x}\ \ \chi_\delta(\mathrm{x})\overline{\psi}^{\mathrm{a},\alpha}(\mathrm{x})\mathrm{V}^{\delta\beta}_{\alpha\gamma}\psi_{\mathrm{a},\beta}(\mathrm{x})\overline{\chi}^{\gamma}(\mathrm{x})\right\}\ . \tag{5}$$

For the free fermion system without $V^{\delta\beta}_{\alpha\gamma}$ interaction, the functional integral becomes

$$Z_0 = \int [\mathrm{D}\psi]\ [\mathrm{D}\overline{\psi}]\ [\mathrm{D}\chi]\ [\mathrm{D}\overline{\chi}]\ \exp\left\{\mathrm{i}\int \mathrm{d}^4\mathrm{x}\left[\overline{\psi}^{\mathrm{a},\alpha}(\mathrm{x})(\partial\!\!\!/)^{\beta}_{\alpha}\psi_{\mathrm{a},\beta}(\mathrm{x}) + \overline{\chi}^{\alpha}(\mathrm{x})(\partial\!\!\!/)^{\beta}_{\alpha}\chi_{\beta}(\mathrm{x})\right]\right\}\ . \tag{6}$$

Let us introduce[14] the dimensional (mass)^3 auxiliary fields denoted by $\Phi^{\gamma}_{\mathrm{a},\alpha}(x)$ and $\overline{\Phi}^{\mathrm{a},\alpha}_{\gamma}(x)$ which represent the fermion-antifermion system, where $\overline{\Phi}$ is defined from Φ as follows:

$$\overline{\Phi}^{\mathrm{a},\alpha}_{\gamma}(x) = (\gamma_0)^{\delta}_{\gamma}\ \Phi^{\dagger \mathrm{a},\beta}_{\delta}(x)\ (\gamma_0)^{\alpha}_{\beta} = (\gamma_0)^{\delta}_{\gamma} \left(\Phi^{\delta}_{\mathrm{a},\beta}(x)\right)^{*} (\gamma_0)^{\alpha}_{\beta}\ .$$

The associated functional integral for these auxiliary fields is

$$Z^{\Phi}_0 = \int [\mathrm{D}\Phi]\ [\mathrm{D}\overline{\Phi}]\quad \exp\left\{-\mathrm{i}\int \mathrm{d}^4\mathrm{x}\ \overline{\Phi}^{\mathrm{a},\alpha}_{\delta}(\mathrm{x}) \mathrm{V}^{\delta\beta}_{\alpha\gamma}\Phi^{\gamma}_{\mathrm{a},\beta}(\mathrm{x})\right\}\ . \tag{7}$$

Now we apply the Hubbard-Stratonovich transformation[13] to the interacting part of the action (3) in the functional integral, and get

$$\exp\left\{\mathrm{i}\int \mathrm{d}^4\mathrm{x}\, \chi_{\delta}(\mathrm{x})\overline{\psi}^{\mathrm{a},\alpha}(\mathrm{x})\mathrm{V}^{\delta\beta}_{\alpha\gamma}\psi_{\mathrm{a},\beta}(\mathrm{x})\overline{\chi}^{\gamma}(\mathrm{x})\right\} = 1\mathrm{Z}^{\Phi}_0 \int [\mathrm{D}\Phi][\mathrm{D}\overline{\Phi}]\ \exp\left\{-\mathrm{i}\int \mathrm{d}^4\mathrm{x}\, \overline{\Phi}^{\mathrm{a},\alpha}_{\delta}(\mathrm{x})\mathrm{V}^{\delta\beta}_{\alpha\gamma}\Phi^{\gamma}_{\mathrm{a},\beta}(\mathrm{x})\right\}$$

$$\times\ \exp\left\{-\mathrm{i}\int \mathrm{d}^4\mathrm{x}\left[\overline{\Delta}^{\mathrm{a},\beta}_{\gamma}(\mathrm{x})\psi_{\mathrm{a},\beta}(\mathrm{x})\overline{\chi}^{\gamma}(\mathrm{x}) + \chi_{\delta}(\mathrm{x})\overline{\psi}^{\mathrm{a},\alpha}(\mathrm{x})\Delta^{\delta}_{\mathrm{a},\alpha}(\mathrm{x})\right]\right\}\ , \tag{8}$$

where

$$\Delta^{\delta}_{\mathrm{a},\alpha}(x) = V^{\delta\beta}_{\alpha\gamma}\Phi^{\gamma}_{\mathrm{a},\beta}(x)\ \ ,\ \ \overline{\Delta}^{\mathrm{a},\beta}_{\gamma}(x) = \overline{\Phi}^{\mathrm{a},\alpha}_{\delta}(x)V^{\delta\beta}_{\alpha\gamma}\ . \tag{9}$$

Physically, the $\Delta^{\delta}_{\mathrm{a},\alpha}(x)$ defined above represents a bosonic field which is a bound state of the fermion-antifermion pair due to the strong interaction $V^{\delta\beta}_{\alpha\gamma}$. It is not necessarily a scalar field and it has the canonical (mass)^1 dimension.

Substituting the expression (8) into the r.h.s. of (5) and integrating out over all the fermionic functional variables $\psi_{\mathrm{a}}, \overline{\psi}^{\mathrm{a}}, \chi, \overline{\chi}$, we can express Z as a functional integral over only the auxiliary fields $\Phi^{\gamma}_{\mathrm{a},\beta}(x)$ and $\overline{\Phi}^{\mathrm{a},\alpha}_{\gamma}(x)$, thus

$$Z = Z_0 Z^{\Phi}_0 \int [\mathrm{D}\Phi]\ [\mathrm{D}\overline{\Phi}]\ \exp\left(\mathrm{i}\mathrm{S}_{\mathrm{eff}}\left[\Phi,\overline{\Phi}\right]\right) \tag{10}$$

with some effective action[14]

$$S_{eff}\left[\Phi,\overline{\Phi}\right] = -\int \mathrm{d}^4\mathrm{x}\ \overline{\Phi}^{\mathrm{a},\alpha}_{\delta}(\mathrm{x})\ \mathrm{V}^{\delta\beta}_{\alpha\gamma}\Phi^{\gamma}_{\mathrm{a},\beta}(\mathrm{x}) + \sum_{\mathrm{n}=1}^{\infty}\mathrm{W}^{(2\mathrm{n})}\left[\Delta,\overline{\Delta}\right]\ , \tag{11}$$

and $W^{(2n)}\left[\Delta,\overline{\Delta}\right]$ is a functional of the n-th order with respect to each kind of fields $\Delta^{\delta}_{\mathrm{a},\alpha}(x)$ and $\overline{\Delta}^{\mathrm{a},\beta}_{\gamma}(x)$. In order to write $W^{(2n)}\left[\Delta,\overline{\Delta}\right]$ in a compact form, we introduce the 4×4 matrices $\widehat{\Delta}_{\mathrm{a}}(x)$ and $\widehat{\overline{\Delta}}^{\mathrm{a}}(x)$ with the elements

$$\left[\widehat{\Delta}_{\mathrm{a}}(x)\right]^{\gamma}_{\alpha} = \Delta^{\gamma}_{\mathrm{a},\alpha}(x)\ \ ,\ \ \left[\widehat{\overline{\Delta}}^{\mathrm{a}}(x)\right]^{\alpha}_{\gamma} = \overline{\Delta}^{\mathrm{a},\alpha}_{\gamma}(x)\ .$$

Then we have

$$W^{(2)}\left[\Delta,\overline{\Delta}\right] = \mathrm{i}\int \mathrm{d}^4\mathrm{x}\ \mathrm{d}^4\mathrm{y}\ \mathrm{Tr}\left[\widehat{\overline{\Delta}}^{\mathrm{a}}(\mathrm{x})\mathrm{S}^{\mathrm{L}}(\mathrm{x}-\mathrm{y})\widehat{\Delta}_{\mathrm{a}}(\mathrm{y})\mathrm{S}^{\mathrm{R}}(\mathrm{y}-\mathrm{x})\right]\ , \tag{12}$$

$$W^{(4)}\left[\Delta,\overline{\Delta}\right] = \mathrm{i}2\int \mathrm{d}^4\mathrm{x}_1\ \mathrm{d}^4\mathrm{y}_1\ \mathrm{d}^4\mathrm{x}_2\ \mathrm{d}^4\mathrm{y}_2\ \mathrm{Tr}\left[\widehat{\overline{\Delta}}^{\mathrm{a}_1}(\mathrm{x}_1)\mathrm{S}^{\mathrm{L}}(\mathrm{x}_1-\mathrm{y}_1)\widehat{\Delta}_{\mathrm{a}_1}(\mathrm{y}_1)\mathrm{S}^{\mathrm{R}}(\mathrm{y}_1-\mathrm{x}_2)\right.$$

$$\times \ \widehat{\overline{\Delta}^{\rm a_2}}(x_2) S^L(x_2-y_2)\widehat{\Delta}_{\rm a_2}(y_2) S^R(y_2-x_1)\Big] \ , \tag{13}$$

$$W^{(2n)}\left[\Delta,\overline{\Delta}\right] = {\rm in}\int {\rm d^4x_1\ d^4y_1\cdots d^4x_n\ d^4y_n\ Tr}\Big[\widehat{\overline{\Delta}^{\rm a_1}}({\rm x_1}){\rm S^L(x_1-y_1)}\widehat{\Delta}_{\rm a_1}({\rm y_1}){\rm S^R(y_1-x_2)}\cdots$$

$$\times \ \cdots\widehat{\overline{\Delta}^{\rm a_n}}(x_n) S^L(x_n-y_n)\widehat{\Delta}_{\rm a_n}(y_n) S^R(y_n-x_1)\Big] \ , \tag{14}$$

where $S^L(x-y)$ and $S^R(x-y)$ being respectively the propagators of left-handed and right-handed massless fermions,

$$\partial\!\!\!/ S^L(x-y) = 1-\gamma_5 2\delta(x-y) \ \ , \ \ \partial\!\!\!/ S^R(x-y) = 1+\gamma_5 2\delta(x-y) \ . \tag{15}$$

From the expression (11) of the auxiliary fields effective action, we derive the field equations

$$\Delta^{\delta}_{{\rm a},\alpha}(x) = V^{\delta\beta}_{\alpha\gamma}\sum_{n=1}^{\infty}\partial\, W^{(2n)}\left[\Delta,\overline{\Delta}\right]\partial\,\overline{\Delta}^{{\rm a},\beta}_{\gamma}(x) \ . \tag{16}$$

The bosonic fields $\Delta^{\delta}_{{\rm a},\alpha}(x)$ and $\overline{\Delta}^{{\rm a},\beta}_{\gamma}(x)$ which describe the quark-antiquark systems bound by the strong interaction $V^{\delta\beta}_{\alpha\gamma}$ must be the solutions of the field equations (16).
Among the most general bosonic field $\Delta^{\gamma}_{{\rm a},\alpha}(x)$, let us consider now a special class of the scalar $\Delta_{\rm a}(x)$ by making the projection

$$\Delta^{\gamma}_{{\rm a},\alpha}(x) = (1+\gamma_5 2)^{\gamma}_{\alpha}\,\Delta_{\rm a}(x) \ ,$$

$$\overline{\Delta}^{{\rm a},\alpha}_{\gamma}(x) = (1-\gamma_5 2)^{\alpha}_{\gamma}\,\Delta^*_{\rm a}(x) \ . \tag{17}$$

In some sense, as we will see, this composite scalar $\Delta_{\rm a}(x)$ substitutes the standard elementary Higgs field to generate masses to both the top quark and the gauge bosons. Associated to this particular $\Delta_{\rm a}(x)$ case, the functionals (12) and (13) become

$$W^{(2)}\left[\Delta,\overline{\Delta}\right] = {\rm i}\int {\rm d^4x\ d^4y}\ \Delta^*_{\rm a}({\rm x})\Delta_{\rm a}({\rm y})\lambda({\rm y-x}) \ , \tag{18}$$

$$W^{(4)}\left[\Delta,\overline{\Delta}\right] = {\rm i2}\int {\rm d^4x\, d^4y\, d^4z\, d^4w}\ \Delta^*_{\rm a}({\rm x})\Delta_{\rm a}({\rm y})$$

$$\times\ \Delta^*_{\rm b}(z)\Delta_{\rm b}(w)\,\Pi(x-y,y-z,z-w) \ , \tag{19}$$

where

$$\lambda(x-y) = {\rm Tr}\left[S^L(x-y)S^R(y-x)\right] \ , \tag{20}$$

$$\Pi(x-y,y-z,z-w) = {\rm Tr}\left[S^L(x-y)S^R(y-z)S^L(z-w)S^R(w-x)\right] \ . \tag{21}$$

For the other functionals $W^{(2n)}\left[\Delta,\overline{\Delta}\right]$, $n>2$, we have similar expressions easily generalized. They are all nonlocal functionals of the scalar fields $\Delta_{\rm a}(x)$ and $\Delta^*_{\rm a}(x)$. Each of them can be expressed in terms of a corresponding local functional of $\Delta_{\rm a}(x)$ and $\Delta^*_{\rm a}(x)$ and their derivatives $\partial^\mu\partial^\nu\cdots\partial^\lambda\Delta_{\rm a}(x)$, $\partial^\mu\partial^\nu\cdots\partial^\lambda\Delta^*_{\rm a}(x)$ to all orders.

In the presence of the vector gauge boson fields $[W_\mu(x)]_{\rm a}^{\rm b}$ and $B_\mu(x)$, the ordinary derivatives ∂_μ must be replaced by the covariant ones D_μ, thus

$$\partial_\mu \Delta_{\rm a}(x) \longrightarrow \mathrm{D}_\mu \Delta_{\rm a}(\mathrm{x}) = \partial_\mu \Delta_{\rm a}(\mathrm{x}) + \mathrm{i}\, [\mathrm{A}_\mu(\mathrm{x})]_{\rm a}^{\rm b}\, \Delta_{\rm b}(\mathrm{x}) \ , \tag{22}$$

$$[A_\mu(x)]_{\rm a}^{\rm b} = g\, [W_\mu(x)]_{\rm a}^{\rm b} + g' 2 B_\mu(x) \delta_{\rm a}^{\rm b} \ . \tag{23}$$

Up to the second order with respect to the first derivatives $\mathrm{D}_\mu \Delta_{\rm a}(\mathrm{x})$, $\mathrm{D}_\mu \Delta_{\rm a}^*(\mathrm{x})$ and the first order with respect to the second derivatives $\mathrm{D}_\mu \mathrm{D}_\nu \Delta_{\rm a}(\mathrm{x})\ ,\ \mathrm{D}_\mu \mathrm{D}_\nu \Delta_{\rm a}^*(\mathrm{x})$, we obtain

$$W^{(2)}\left[\Delta, \overline{\Delta}\right] = \mathrm{i} \int \mathrm{d}^4\mathrm{x} \left\{ \lambda^{(0)}(\mathrm{x}) \Delta_{\rm a}^*(\mathrm{x}) \Delta_{\rm a}(\mathrm{x}) + 12 \lambda^{(2)}(\mathrm{x}) \Delta_{\rm a}^*(\mathrm{x}) \mathrm{D}^\mu \mathrm{D}_\mu \Delta_{\rm a}(\mathrm{x}) \right\} \ , \tag{24}$$

$$W^{(4)}\left[\Delta, \overline{\Delta}\right] = \mathrm{i}2 \int \mathrm{d}^4\mathrm{x} \left\{ \Pi^{(0)}(\mathrm{x}) \Delta_{\rm a}^*(\mathrm{x}) \Delta_{\rm a}(\mathrm{x}) \Delta_{\rm b}^*(\mathrm{x}) \Delta_{\rm b}(\mathrm{x}) \right.$$

$$\left. + \Pi^{(2)}(x) \Delta_{\rm a}^*(x) \Delta_{\rm a}(x) \Delta_{\rm b}^*(x) D^\mu D_\mu \Delta_{\rm b}(x) + \Omega(x) \Delta_{\rm a}^*(x) D^\mu \Delta_{\rm a}(x) \Delta_{\rm b}^*(x) D_\mu \Delta_{\rm b}(x) \right\} \ , \tag{25}$$

where

$$\lambda^{(0)}(x) = \int \mathrm{d}^4\mathrm{y}\, \lambda(\mathrm{x} - \mathrm{y}) \ , \tag{26}$$

$$\lambda^{(2)}(x) = \int \mathrm{d}^4\mathrm{y}\, (\mathrm{y} - \mathrm{x})^\mu\, (\mathrm{y} - \mathrm{x})_\mu \lambda(\mathrm{x} - \mathrm{y}) \ , \tag{27}$$

$$\Pi^{(0)}(x) = \int \mathrm{d}^4\mathrm{y}\, \mathrm{d}^4\mathrm{z}\, \mathrm{d}^4\mathrm{w}\, \Pi(\mathrm{x} - \mathrm{y}, \mathrm{y} - \mathrm{z}, \mathrm{z} - \mathrm{w}) \ , \tag{28}$$

$$\Pi^{(2)}(x) = \int \mathrm{d}^4\mathrm{y}\, \mathrm{d}^4\mathrm{z}\, \mathrm{d}^4\mathrm{w}\, (\mathrm{y} - \mathrm{x})^\mu (\mathrm{y} - \mathrm{x})_\mu \Pi(\mathrm{x} - \mathrm{y}, \mathrm{y} - \mathrm{z}, \mathrm{z} - \mathrm{w}) \ , \tag{29}$$

$$\Omega(x) = \int \mathrm{d}^4\mathrm{y}\, \mathrm{d}^4\mathrm{z}\, \mathrm{d}^4\mathrm{w}\, (\mathrm{y} - \mathrm{x})^\mu (\mathrm{w} - \mathrm{z})_\mu \Pi(\mathrm{x} - \mathrm{y}, \mathrm{y} - \mathrm{z}, \mathrm{z} - \mathrm{w}) \ . \tag{30}$$

For $W^{(2n)}\left[\Delta, \overline{\Delta}\right]$ with $n > 2$, we have expressions generalizing (24) and (25).

Now in both sides of the field equations (16), as well as in (24), (25) and their generalizations for $W^{(2n)}\left[\Delta, \overline{\Delta}\right], n > 2$, we set the scalar composite fields $\Delta_{\rm a}(x)$ to be equal to their vacuum expectation values $\Delta_{\rm a}^{(0)}$ which describe the condensate of quark-antiquark pairs. From (1) and (2), we note that among the two components $\Delta_{\rm a}^{(0)}$ of the doublet, only the second component $\Delta_{\rm a=2}^{(0)}$ is a neutral field and could have non-vanishing vacuum expectation value, thus

$$\Delta_{\rm a}^{(0)} = \delta_{\rm a2} \Delta \tag{31}$$

where Δ is some real constant that we put equal to the top mass $\Delta = M_t$. This point can be seen from the term $\overline{\Delta}_\gamma^{\rm a,\beta}(x) \psi_{\rm a,\beta}(x) \overline{\chi}^\gamma(x) + \chi_\delta(x) \overline{\psi}^{\rm a,\alpha}(x) \Delta_{\rm a,\alpha}^\delta(x)$ figured on the second line of (8). With this constant value (31) of the scalar fields $\Delta_{\rm a}(x)$, the field equation (16) reduces to

$$14G = N_c{-}\mathrm{i}(2\pi)^4 \int \mathrm{d}^4\mathrm{p}\mathrm{p}^2 - \mathrm{M}_\mathrm{t}^2 + \mathrm{i}\epsilon \ , \tag{32}$$

where the number of quark colors $N_c = 3$ is taken into account in the traces of (20), (21). The divergence of the integral in the r.h.s. of (32) may be handled by a momentum cutoff Λ written in a covariant manner, and we obtain the algebraic equation[15]

$$N_c 14\pi^2 \int_0^{\Lambda^2} x\, \mathrm{dxx} + \mathrm{M_t^2} = N_c 4\pi^2 \left[\Lambda^2 - M_t^2 \,\ln\Lambda^2 + M_t^2 M_t^2\right] = 1G\ . \tag{33}$$

The solution of this equation does exist if and only if Λ satisfies the condition

$$\Lambda^2 > 4\pi^2 G N_c\ . \tag{34}$$

From (33), let us denote by $y = f(x)$ the positive solution of the equation $\ln(1+y) = xy$ in the interval $0 < x < 1$, then we have

$$M_t^2 = \Lambda^2 f(x_0)\ \ ,\ x_0 = 1 - 4\pi^2 G N_c \Lambda^2\ . \tag{35}$$

This equation, reminiscent of [15], determines M_t^2 in terms of Λ^2 and G.

In order to derive the gauge boson masses, we substitute the constant value (31) of the scalar fields $\Delta_{\mathrm{a}}(x)$ into the expressions of $W^{(2n)}\left[\Delta, \overline{\Delta}\right]$, sum up all these expressions and separate out the quadratic term $A_\mu A^\mu$ which contributes to the vector gauge boson mass in the effective Lagrangian $\mathcal{L}^A_{mass}(x)$. We finally obtain

$$\mathcal{L}^A_{mass}(x) = -12\left\{M_t^2\,\mathcal{I}\ [A_\mu(x)]^{\mathrm{c}}_2\,[A^\mu(x)]^2_{\mathrm{c}} + M_t^4 \mathcal{J}\,[A_\mu(x)]^2_2\,[A^\mu(x)]^2_2\right\}\ , \tag{36}$$

where

$$\mathcal{I} = -\mathrm{i}(2\pi)^4 \int \mathrm{d^4pTr}\left[\mathrm{\widetilde{S}^L(p)\widetilde{S}^R(p)\widetilde{S}^L(p)\widetilde{S}^R(p)}\right] 1 - \mathrm{M_t^2 p^2}\ , \tag{37}$$

$$\mathcal{J} = -12\mathrm{i}(2\pi)^4 \int \mathrm{d^4pTr}\left[\mathrm{\widetilde{S}^L(p)\widetilde{S}^R(p)\widetilde{S}^L(p)\widetilde{S}^R(p)\widetilde{S}^L(p)\widetilde{S}^R(p)}\right]\left(1 - \mathrm{M_t^2 p^2}\right)^2\ . \tag{38}$$

$\widetilde{S}^L(p)$ and $\widetilde{S}^R(p)$ being the propagators of the free left-handed and right-handed massless fermions in momentum space:

$$\widetilde{S}^L(p) = 1 - \gamma_5 2\ \mathrm{i}\ \not{p}\mathrm{p}^2 + \mathrm{i}\epsilon\ \ ,\ \ \widetilde{S}^R(p) = 1 + \gamma_5 2\ \mathrm{i}\ \not{p}\mathrm{p}^2 + \mathrm{i}\epsilon\ . \tag{39}$$

Since $\mathcal{I}$ is divergent, we again use the cutoff Λ in a covariant manner and get

$$\mathcal{I} = N_c 8\pi^2 \int_0^{\Lambda^2} \mathrm{dxx} + \mathrm{M_t^2} = N_c 8\pi^2 \ln\Lambda^2 + M_t^2 M_t^2\ . \tag{40}$$

The integral $\mathcal{J}$ is convergent and equals

$$\mathcal{J} = -N_c 16\pi^2 \int_0^{\infty} \mathrm{dx(x + M_t^2)^2} = -N_c 16\pi^2 1 M_t^2\ . \tag{41}$$

Setting

$$(W_\mu)^1_2 = 1\sqrt{2}W^+_\mu\ \ ,\ \ (W_\mu)^2_1 = 1\sqrt{2}W^-_\mu\ , \tag{42}$$

$$(W_\mu)^1_1 = -(W_\mu)^2_2 = 12\left[\sin\theta_W A_\mu + \cos\theta_W Z_\mu\right]\ , \tag{43}$$

$$B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu \ , \tag{44}$$

and using $g\sin\theta_W - g'\cos\theta_W = 0$, we obtain from (36)

$$\mathcal{L}^A_{mass}(x) = -M_W^2\, W^+_\mu(x) W^\mu(x) - 12 M_Z^2\, Z_\mu(x) Z^\mu(x) \ , \tag{45}$$

where the gauge boson masses are found to be

$$M_W^2 = N_c g^2 32\pi^2 M_t^2 \ln\Lambda^2 + M_t^2 M_t^2 \ , \tag{46}$$

$$M_Z^2 = N_c g^2 + g^{'2} 32\pi^2 M_t^2 \left[\ln\Lambda^2 + M_t^2 M_t^2 - 12\right] \ . \tag{47}$$

Equation (46) is reminiscent of the Pagel-Stokar formula[16] which relates the charged pion decay constant $f_\pi \approx \ 131$ MeV to the dynamically generated mass obtained in QCD for the up down quarks. From QCD to electroweak interactions, the f_π of the former is replaced by the vacuum expectation value $v = [\sqrt{2}G_{\rm F}]^{-1/2} = 2M_W/g \approx 246$ GeV of the latter. This may be schematically transcribed as:

In QCD, the Pagel-Stokar formula relates f_π to $m_{\rm u,d}$. In dynamical symmetry breaking of the electroweak interactions, equation (46) gives v in terms of M_t.

Furthermore, with $g^2 + g^{'2} = g^2/\cos^2\theta_W$, we get from (46) and (47)

$$\rho \equiv M_W^2 M_Z^2 \cos^2\theta_W = 1 + 12\ln\left(\Lambda^2 M_t^2 + 1\right) \ . \tag{48}$$

The precision electroweak measurements at the 10^{-3} level force the scale Λ of top-condensation models to be very high about 10^{15} GeV when we compare experimental data with (46), (47) and (48).

It is gratifying to note that using very different methods, we recover some results previously obtained in the literature [7, 15, 16]. We also remark that in the standard Higgs mechanism, at the tree-level the parameter ρ is equal to unity. The correction $\Delta\,\rho = |\rho - 1|$ can only come from higher order loops to which the top quark contribution at order g^2 is quadratic in its mass M_t, thus[17]

$$\Delta\rho = 3g^2 64\pi^2 \cos^2\theta_W\, M_t^2 M_W^2 \ . \tag{49}$$

The result (49) is in sharp contrast with the dynamical symmetry model discussed here for which the ρ parameter is already different from unity to zero order of the coupling constants g, g', as shown by equation (48). The unbroken global symmetry of the Higgs sector (translated by $\rho = 1$) usually called custodial $SU(2)$ symmetry[18] of the electroweak theory is systematically violated by a smooth logarithm of M_t. The patterns of custodial $SU(2)$ symmetry breaking as illustrated by (48) and (49) are conceptually not the same.

We now briefly discuss some phenomenological consequences of the top-condensation models, in particular the production and decay modes of the neutral scalar denoted by H_t which replaces the elementary Higgs boson H^0 of the standard model.

1- In general, whatever the schema invoked to dynamically generate the top and the W, Z masses, there must exist a triplet of new Nambu-Goldstone bosons resulting from the breaking of chiral symmetry in the top-bottom system postulated in (1) $\cdots$ (4). One set is absorbed by the W, Z to acquire masses as shown in (46), (47) and the other the so called top-pion remains. Furthermore, a neutral CP-even state analogous to the σ boson in QCD must exist, it is a scalar $\bar{t}t$ bound state denoted by H_t. Since the force postulated in (4) that ties top-antitop pair is so strong that it can dynamically generate such a huge 175 GeV mass, it is likely that the biding energy is large in H_t and its mass could be smaller[10] than twice[5] the top mass. This is taken as a very rough indication of where to find H_t.

2- The production cross section of H_t is governed by gluon-gluon fusion[17] into $t\bar{t}$ pairs, so that hadron colliders at the FermiLab Tevatron and the Cern LHC hadron colliders are appropriately the right places for H_t searches. On the other hand, the $Z+H_t$ associated production by lepton colliders, $e^+e^- \to Z^* \to Z+H_t$ is largely suppressed, since direct coupling ZZH_t is absent. This is also in sharp contrast with the elementary Higgs boson H^0 production dominated[17] by the reaction $e^+e^- \to Z^* \to Z+H^0$, because direct coupling ZZH^0 is large.

3- Due to the nature of a strongly bound $\bar{t}t$ state, the H_t is leptophobic, and "almost" hadrophobic, i.e. it cannot decay into leptons and "light" hadrons made up by the first two families up, down, charm, strange quarks at the tree level (lowest order of the coupling constants g, g'). Indeed, the Yukawa direct couplings of H_t with leptons and the first two families of quarks are absent, contrarily to the standard elementary Higgs boson H^0 case. Therefore the H_t although so massive would have a very narrow width (only a few GeV) in sharp contrast with the standard elementary Higgs boson H^0 which has a width[17] around 18 GeV for a hypothetical 350 GeV $\approx 2M_t$ mass. The decays of H_t into "light" hadrons can only proceed through gluons emission, similarly to quarkonium J/ψ and Υ decays which are suppressed by the Okubo-Zweig-Iizuka (OZI) rule reflecting QCD asymptotic freedom.

4- The most distinctive signatures of H_t would be its dominant decay modes into the bottomonium Υ and an energetic photon or Υ accompanied by the Z weak boson, as depicted by Fig.1. This comes from the special situation of the third family top-bottom quarks which have the additional topcolor interaction – mediated by topgluons $\mathcal{G}_t$ – thus destroying the universality between the three quark families. Due to the non-universal character of the top-bottom system which does not possess the Glashow-Iliopoulos-Maiani (GIM) cancelation, flavor changing neutral decay of H_t into $t\bar{c}+\bar{t}c$ channels is another spectacular signature[19] of the top condensation models.

The ratio $\Gamma(H_t \to \Upsilon + Z)/\Gamma(H_t \to \Upsilon + \gamma)$ is found to be

$$\Gamma(H_t \to \Upsilon + Z)\Gamma(H_t \to \Upsilon + \gamma) = (34\sin\theta_W\cos\theta_W)^2\left[\left(12-43\sin^2\theta_W\right)^2+14\right]\left(1-M_Z^2 M_{H_t}^2\right) \approx 0.8\ . \tag{50}$$

Finally we remark that hadronic decays of H_t must proceed into $\mathrm{b}\overline{\mathrm{b}}$ pair through two topgluons $\mathcal{G}_t$ exchange in Fig.2. These dominant decay modes into B^*, B mesons $H_t \to B^*(B) + \overline{B}^*(\overline{B})$ are OZI unsuppressed.

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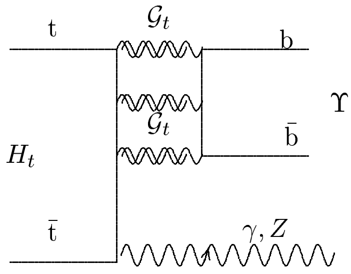

Fig.1

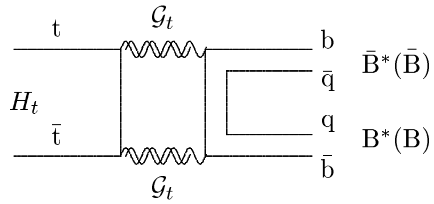

Fig.2

Figure Captions

Fig.1 $H_t \rightarrow \Upsilon + Z(\gamma)$

Fig.2 The OZI unsuppressed $H_t \rightarrow \overline{\mathrm{B}}^*(\overline{\mathrm{B}}) + \mathrm{B}^*(\mathrm{B})$ by topgluons $\mathcal{G}_t$ exchange